

## Division of Fractions

A. This is based on the knowledge that dividing by a number is equivalent to multiplying by its reciprocal and the commutative, and associative laws. Last it needs to be understood that a fraction can be thought of as the quotient of a division (e.g.  $\frac{1}{2} = 1 \div 2$ ), see grade 5 common core standards.

Let's investigate  $1\frac{3}{4} \div \frac{1}{2}$

$$1\frac{3}{4} \div \frac{1}{2} = 1\frac{3}{4} \div (1 \div 2), \quad \text{interpreting the divisor as a quotient}$$

$$1\frac{3}{4} \div (1 \div 2) = 1\frac{3}{4} \div 1 \times 2, \quad \text{distributing the division (like we would with } -(1-2) = -1+2)$$

$$1\frac{3}{4} \div 1 \times 2 = 1\frac{3}{4} \times 2 \div 1, \quad \text{in the order of operations multiplication and division can occur in either order.}$$

$$1\frac{3}{4} \times 2 \div 1 = 1\frac{3}{4} \times (2 \div 1), \quad \text{factoring out the multiplication}$$

And now we are done! Just a little clean-up:

$$1\frac{3}{4} \times (2 \div 1) = 1\frac{3}{4} \times \frac{2}{1}, \quad \text{Calculating the quotient.}$$

B. An alternative approach that is based on equivalence:

$$\begin{aligned} 1\frac{3}{4} \div \frac{1}{2} &= (1\frac{3}{4} \times \frac{2}{1}) \div (\frac{1}{2} \times \frac{2}{1}) \\ &= (1\frac{3}{4} \times \frac{2}{1}) \div 1 \\ &= 1\frac{3}{4} \times \frac{2}{1} \end{aligned}$$

C. Working with Mixed numbers using the distributive law

C1. First rewrite division as multiplication by its reciprocal and expand the mixed number:

$$\begin{aligned} 1\frac{3}{4} \div \frac{1}{2} &= (1 + \frac{3}{4}) \times 2 \\ &= (1 \times 2) + (\frac{3}{4} \times 2) \\ &= 2 + 1\frac{1}{2} \\ &= 3\frac{1}{2} \end{aligned}$$

C2. Expand the mixed number and then distribute the division, then reciprocate and multiply:

$$\begin{aligned} 1\frac{3}{4} \div \frac{1}{2} &= (1 + \frac{3}{4}) \div \frac{1}{2} \\ &= (1 \div \frac{1}{2}) + (\frac{3}{4} \div \frac{1}{2}) \\ &= (1 \times 2) + (\frac{3}{4} \times 2) \\ &= 2 + 1\frac{1}{2} = 3\frac{1}{2} \end{aligned}$$

D. Dividing fractions in the same manner as we multiply them (divide numerators and denominators):

$$1\frac{3}{4} \div \frac{1}{2} = \frac{7}{4} \div \frac{1}{2}, \quad \text{Write the mixed number as an “improper” fraction}$$

$$\frac{7}{4} \div \frac{1}{2} = \frac{7 \div 1}{4 \div 2} = \frac{7}{2} = 3\frac{1}{2}, \quad \text{Divide the numerators and the denominators, write as mixed number.}$$

Proof:

$$\begin{aligned} \frac{a}{b} \div \frac{c}{d} &= b\left(\frac{a}{b}\right) \div b\left(\frac{c}{d}\right) & \frac{8}{15} \div \frac{2}{3} &= 15\left(\frac{8}{15}\right) \div 15\left(\frac{2}{3}\right) \\ &= a \div \frac{bc}{d} & &= 8 \div \frac{15 \times 2}{3} \\ &= a \div \left(c \times \frac{b}{d}\right) & &= 8 \div \left(2 \times \frac{15}{3}\right) \\ &= a \div c \div \frac{b}{d} & &= 8 \div 2 \div \frac{15}{3} \\ &= (a \div c) \div (b \div d) & &= (8 \div 2) \div (15 \div 3) \\ &= \frac{a \div c}{b \div d} & &= \frac{8 \div 2}{15 \div 3} \end{aligned}$$